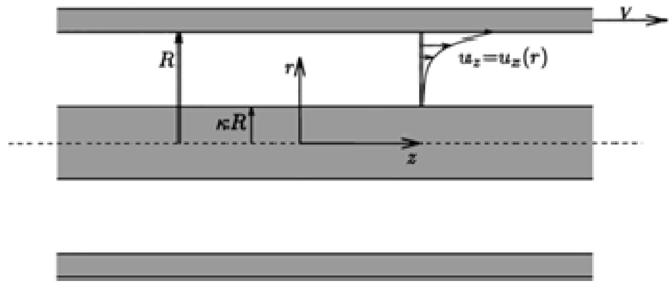
Sahand University of Technology
Chemical Engineering Faculty
Instructor: Dr. Alizadeh

Advanced Fluid Mechanics HW3
Return date
Spring Semester

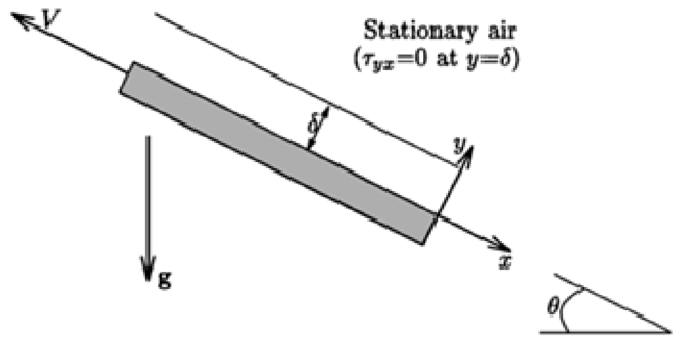
1. By selecting an appropriate element and using shell momentum balance, find the velocity distribution, volumetric flow rate and shear stress function for Flow in an annulus driven by the motion of the outer cylinder.



2. Consider flow of a thin, uniform film of an incompressible Newtonian liquid on an infinite, inclined plate that moves upwards with constant speed V, as shown in Fig. The ambient air is assumed to be stationary, and the surface tension is negligible.

(a) By using sell momentum balance calculate the velocity $u_x(y)$ of the film in terms of V, δ, ρ, μ, g and θ.

(b) Calculate the speed V of the plate at which the net volumetric flow rate is zero.



3. The laminar flow velocity profile in a pipe for a power law liquid in steady state flow is given by the equation:

$$v_x = u\left(\frac{3n+1}{n+1}\right)\left[1-\left(\frac{2r}{d_i}\right)^{(n+1)/n}\right]$$

where n is the power law index and u is the mean velocity. Use this to derive the expression:

$$\left(\frac{-dv_x}{dr}\right) = \left(\frac{8u}{d_i}\right)\left(\frac{3n+1}{4n}\right)$$

for the velocity gradient at the pipe wall.

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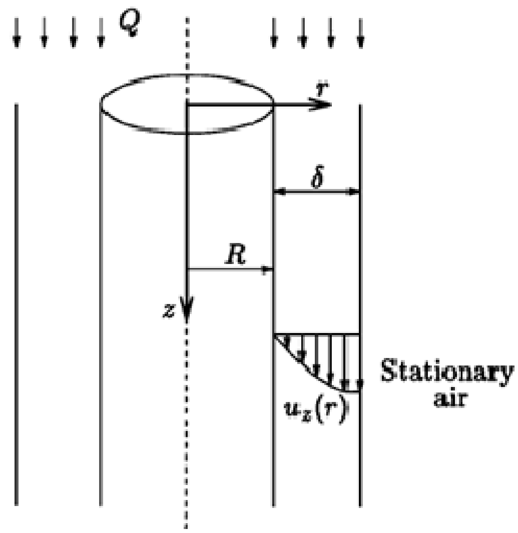
4. By selecting an appropriate element and using shell momentum balance, find the velocity distribution, volumetric flow rate and shear stress function for fully developed steady film flow down a vertical cylinder. How does the film thickness relate to the total film volumetric flow rate Q?

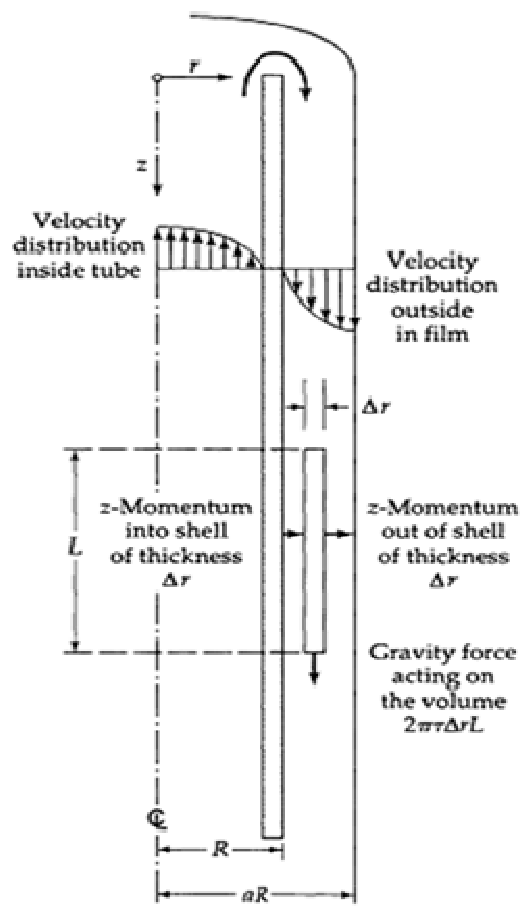


5. In a gas absorption experiment a viscous fluid flows upward through a small circular tube and then downward in laminar flow on the outside. Set up a momentum balance over a shell of thickness Δr in the film, As shown in figure

Note that ‘momentum in’ and ‘momentum out’ arrows are always taken in the positive coordinate direction, even though in this problem the momentum is flowing through the cylindrical surface in the negative r direction.
(a) Show that the velocity distribution in the falling film (neglecting end effect) is

$$v_z = \frac{\rho g R^2}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2 + 2a^2\left(\frac{r}{R}\right)\right]$$

(b) Obtain an expression for the mass rate of flow in the film.

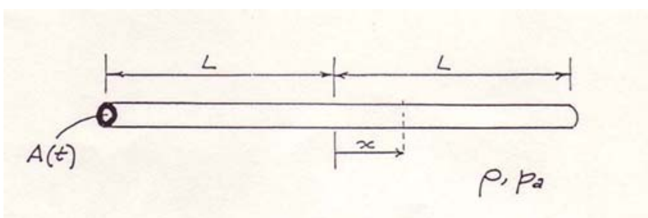


6. A small tube of length 2L is submerged in a pool of liquid (density ρ, pressure P_a). the tube is open at both ends, and filled with liquid. However, it is made of piezoelectric material, and its cross-sectional area A (which is uniform over the tube's length) can be controlled by the application of an electric voltage. Suppose that, by the application of a suitable voltage, the tube area is reduced in time according to specified function A(t), which is monotonically decreasing. Assuming that the flow inside the tube is incompressible and inviscid, obtain in terms of the given quantities and the function A(t) and its derivatives, expressions for:

(a) The flow speed u at a station in the tube

(b) The pressure at the tune's center point, x=0

(c) Are your results in (a) and (b) valid for increasing as well as decreasing A(t)?



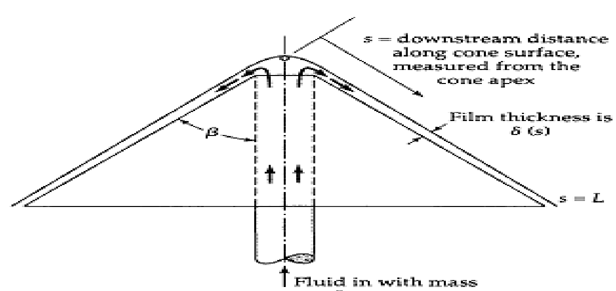
7- A fluid flows upward through a circular tube and then downward on a conical surface. Find the film thickness as a function of the distance s down the cone.

Assume that the results flow of falling film apply approximately over any small region of the cone surface. Show that a mass balance on a ring of liquid contained between s and s + Δs gives:

$$\frac{d}{ds}\left(s\delta\langle v\rangle\right) = 0 \text{ or } \frac{d}{ds}\left(s\delta^3\right) = 0$$

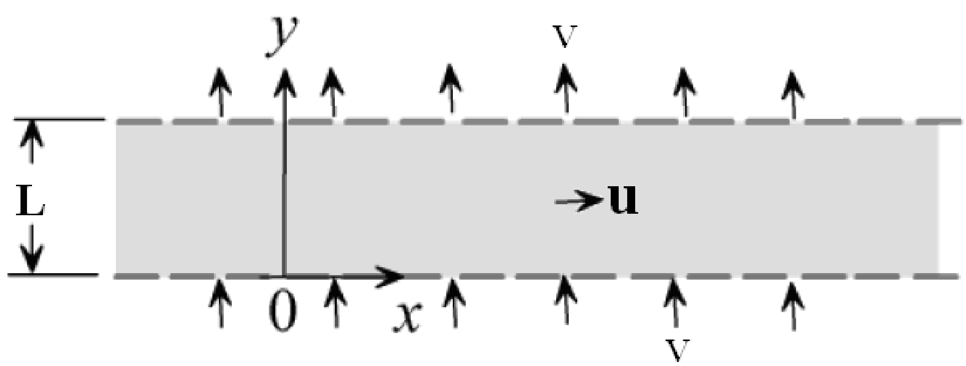
Integrate this equation and evaluate the constant of integration by equating the mass rate of flow ω up the central tube to that flowing down the conical surface at s=L. obtain the flowing expression for the film thickness:

$$\delta = \sqrt[3]{\frac{3\mu\omega}{\pi\rho^2 g L \sin(2\beta)}\left(\frac{L}{s}\right)}$$

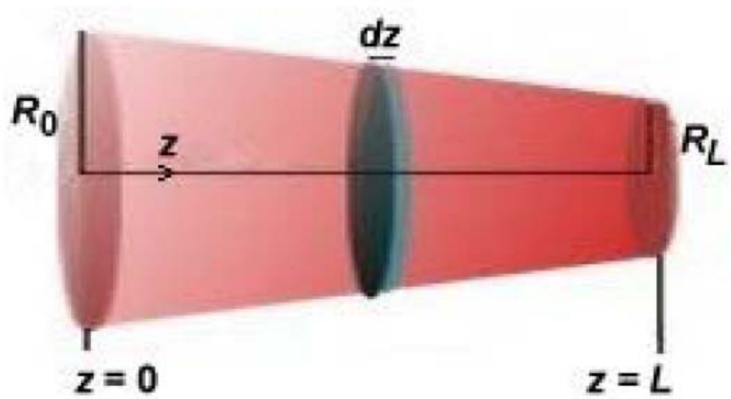


8-Two large porous plates are separated by a distance L. An incompressible constant-property fluid fills the channel formed by the plates; also plates are fixed. An axial pressure gradient dp/dx is applied to the fluid to set it in motion. The same fluid is also injected through the lower plate with a normal velocity V. Fluid is removed along the upper plate at velocity V. The injected fluid is identical to the channel fluid. By Neglecting gravity, If the velocity in x direction (u) depends upon only y. **(a)-** using continuity and Navier- Stokes equation find velocity in x direction (u).

(b)- find the viscous drag at the bottom plate



9-A fluid (of constant density ρ) is in incompressible, laminar flow through a tube of length L. The radius of the tube of circular cross section changes linearly from R_0 at the tube entrance (z = 0) to a slightly smaller value R_L at the tube exit (z = L). Determine the mass flow rate vs. pressure drop (w vs. ΔP)relationship for a Newtonian fluid (of constant viscosity μ).(Hint:use Hagen-Poiseuille equation)



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Consider a fluid (of density ρ) in incompressible, laminar flow in a plane narrow slit of length L and width W formed by two flat parallel walls that are a distance $2B$ apart. End effects may be neglected because $B << W << L$. The fluid flows under the influence of a pressure difference Δp, gravity or both.

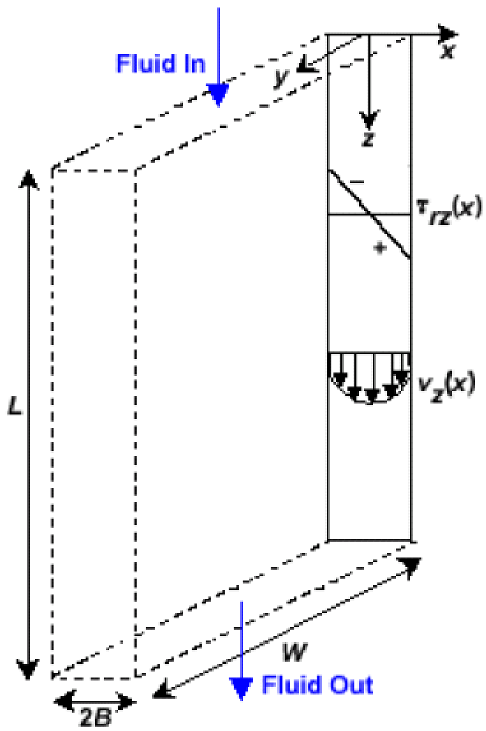

Figure. Fluid flow in plane narrow slit.

a) Determine the steady-state velocity distribution for a non-Newtonian fluid that obeys the power law model (e.g., a polymer liquid).

b) Obtain the mass flow rate for a power law fluid in slit flow.